

# The Determination of the CKM Matrix

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## Abstract

A general discussion of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is given and the importance stressed of determining the matrix elements as an essential part of understanding CP violation in and beyond the Standard Model. The status of knowledge of the matrix elements connecting the first and second generation quarks is reviewed. A perspective on determinations of the full CKM matrix is presented as an introduction to the separate contributions to the panel discussion that follows.

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# 1 Introduction

In the Standard Model with  $SU(2) \times U(1)$  as the gauge group of electroweak interactions, both the quarks and leptons are assigned to be left-handed doublets and right-handed singlets. The quark mass eigenstates differ from the weak eigenstates, and the matrix relating these bases was defined for six quarks and given an explicit parametrization by Kobayashi and Maskawa [1] in 1973. It generalizes the four-quark case, where the matrix is parametrized by a single angle, the Cabibbo angle [2].

By convention, the mixing is usually expressed in terms of a  $3 \times 3$  matrix  $V$  operating on the charge  $-e/3$  quark mass eigenstates ( $d$ ,  $s$ , and  $b$ ):

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (1)$$

The matrix  $V$  is unitary, and this Cabibbo-Kobayashi-Maskawa (CKM) matrix can physically be fully specified by four real parameters. These can be taken to be three “rotation” angles and one phase. CP violation has a natural place and occurs if the phase is not  $0^\circ$  or  $180^\circ$  and the other angles are not  $0^\circ$  or  $90^\circ$ , *i.e.*, if there is mixing between each pair of generations of quarks and there is a non-trivial phase.

We know experimentally that all three angles that characterize the CKM matrix are small but non-zero. There is an expectation that the single non-trivial phase should be non-zero as well. (We will return at the end to the status of showing whether the phase is non-zero.) If CP violation arises from the CKM matrix, there is both a natural scale and a special pattern for CP-violating effects (and for flavor-changing-neutral-current effects more generally). These are made manifest in the theoretical predictions for various decay and mixing processes for B mesons found throughout the Proceedings of this conference.

The major outstanding question with regard to CP violation is no longer what was raised at conferences for many years, namely “What is the origin of CP violation?” We have an origin in the CKM matrix of the Standard Model. It is not unreasonable that this accounts for most of the CP-violating effects observed to date and to be observed in the near future. Rather, the question to be answered by experiment and theory in the coming decade is: “Are there CP-violating effects that do not arise from the CKM matrix and instead come from physics beyond the Standard Model.”

Thus we expect that the situation we will soon find ourselves in is one where the largest measured CP-violating effects arise from the Standard Model, with possible small contributions from new physics. Consequently, to establish the existence of new physics effects we will need to know the Standard Model effects accurately. Fortunately, as the discussion that follows shows, we are moving into an era of “precision” CKM measurements. This will hopefully give us the elements of the CKM matrix with sufficient accuracy that, together with improved theoretical calculations of hadronic matrix elements, we will be able to pin down the Standard Model contributions and establish the presence of possible new physics.

## 2 The Large CKM Matrix Elements

The other members of the panel discussion on the CKM matrix, M. Artuso [3], P. Faccioli [4], J. Rosner [5], and A. Stocchi [6], have concentrated their contributions on analyses of the small CKM matrix elements involving the third generation b-quark and t-quark or to making overall fits to the whole matrix. In this section I discuss some of the developments involving the CKM matrix elements connecting the first and second generation quarks. These follow closely the review by K. Kleinknecht, B. Renk, and myself in the Review of Particle Physics [7]. Detailed references can be found there.

- The element  $|V_{ud}|$  has been most accurately determined through analysis of nuclear beta decays that involve transitions between states with zero spin for which only the weak vector current contributes. Taking account of higher order radiative corrections is essential, and the remaining debate centers on these corrections and on whether there is a change in charge-symmetry violation for quarks inside nuclear matter at the tenths of a percent level. Taking both these uncertainties, a value of  $|V_{ud}| = 0.9740 \pm 0.0010$  is quoted [7]. While the above has been the standard method to obtain  $|V_{ud}|$  for years, recently there has been an improvement in precision of the value obtained from neutron decays. This has fewer theoretical uncertainties, but relies on both the value of  $g_A/g_V$  and on the neutron lifetime. Experimental progress has been made on the former quantity using very highly polarized cold neutrons together with improved detectors. This results in  $|V_{ud}| = 0.9728 \pm 0.0012$  from neutron decay, and averaging the two independent results for  $|V_{ud}|$  gives

the value  $|V_{ud}| = 0.9735 \pm 0.0008$  quoted in Ref. [7]. This is about two sigma lower than expected from unitarity of the first row of the CKM matrix, and therefore bears watching.

- The matrix element  $|V_{us}|$ , the sine of the Cabibbo angle, is best determined from analysis of  $K_{e3}$  decays, which yield the value  $|V_{us}| = 0.2196 \pm 0.0023$ . Analysis of hyperon decays has larger theoretical uncertainties and gives a result for  $|V_{us}|$  that is not inconsistent. Given the progress in experimental techniques and our interest in a more precise check of unitarity, a more modern experiment and analysis would be quite worthwhile.
- In principle, one could determine  $|V_{cd}|$  from charm decays to non-strange particles, but we lack both high statistics data and accurate theoretical input on the relevant form factors. The most accurate value presently comes from neutrino and antineutrino production of charm off valence  $d$  quarks. This yields  $|V_{cd}| = 0.224 \pm 0.016$ .
- Values of  $|V_{cs}|$  can be obtained from neutrino production of charm, but they are dependent on assumptions about the strange-quark density in the parton sea. More accurate values can be obtained from charm decays to strange particles, and in particular  $D \rightarrow \bar{K}e^+\nu_e$ . Here the primary source of error is in the theoretical estimation of the associated form factor and leads to the value  $|V_{cs}| = 1.04 \pm 0.16$ . Significant progress here has recently come from the high energy regime at LEP, where direct measurements [8] of  $|V_{cs}|$  in charm-tagged  $W$  decays give  $|V_{cs}| = 0.97 \pm 0.09$  (stat.)  $\pm 0.07$  (syst.). This new technique already gives a value with a comparable error bar to that from  $D$  decays. Furthermore, the  $W$  decays into all possible pairs of first and second generation quark-antiquark pairs, weighted by the squares of the relevant CKM matrix elements. The result [9] from LEP is that  $\sum_{i,j} |V_{ij}|^2 = 2.032 \pm 0.032$ , where the sum extends over  $i = u, c$  and  $j = d, s, b$ . Since five of the six CKM matrix elements involved are well measured or contribute negligibly to the sum of the squares, this measurement can also be used to obtain a precision measurement of  $|V_{cs}| = 0.9891 \pm 0.016$ . The error bar has been reduced by an order of magnitude from that obtained using charm decays!

### 3 Overall Determination of the CKM Matrix

Looking back at the present limits on the accuracy in determining the large CKM matrix elements, we quickly see that it is primarily not a question of experimental statistics. Rather, it is the systematics and especially the “theoretical systematics” that limit us. This becomes even more obvious when we consider the determination of  $|V_{ub}|$ ,  $|V_{cb}|$ , and  $|V_{td}|$ . Time and again we need theoretical calculations of matrix elements or parameters that relate weak amplitudes at the quark level to those at the hadron level, whether it be for inclusive or exclusive decay modes.

Theoretical errors in such cases are hard to estimate. It is one thing to vary the parameters that enter a given calculation over a reasonable range and thereby deduce how the final result will vary within a given model. This is usually easy to do and is a minimal estimate of the potential error. It is quite another matter to estimate the effects of what has been left out of the model or theory, or to know the accuracy of an *ab initio* assumption such as quark-hadron duality in a new situation.

Furthermore, such errors are generally not Gaussian. It still makes sense to quote a reasonable range for a given CKM matrix element that corresponds roughly to “1  $\sigma$ ” or to “90% confidence level,” but combining several such measurements should be done with great care. As was pointed out during the panel discussion, this can have profound physics consequences in making an overall fit to the CKM matrix, as those with “small” errors would say that we are already forced to a non-trivial unitarity triangle of  $V_{ub}^*$ ,  $V_{td}$ , and  $\sin\theta_{12}V_{cb}$  in the complex plane (and therefore a non-trivial phase in the CKM matrix and CP-violation in the Standard Model) just from knowledge of the lengths of the triangle’s sides. *Caveat emptor*.

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